THE OPTIMAL TIMING OF URBAN LAND DEVELOPMENT

DONALD C. SHOUP, University of California,
Los Angeles

While economists have devoted much attention to the problem of allocating urban land among competing uses, much less attention has been given to the problem of the *timing* of the development of land from rural to urban use or from a lower to a higher urban use. Such development and redevelopment of land continually take place in and around urban areas, and usually considerable land is temporarily held idle or in a low-valued use, awaiting development to a higher use. Land awaiting new development has typically been said to be "ripening" for a higher use.[1] The purpose of this paper is to go more deeply into the question of the optimal timing of the development of land from a lower to a higher use and to clarify some of the important considerations that a private investor or a government should take into account when making land use decisions.

Some observations on the role that speculators play in bringing about optimal development timing decisions in the real estate market also emerge as a by-product of the analysis.

THE STATIC CASE

In a static situation, the criterion for the profit maximizing allocation of land among different uses is that value of marginal product in all uses should be equal. If the marginal product of land in one use is higher than in another, land should be shifted from the lower into the higher valued use, up to the point where the two marginal products are equal. Since land yields its service through time, this statement must be taken in a present value sense; that is, the present discounted value of the marginal product of land in all uses should be equal.[2]

The value of the marginal product of an urban site is determined by the production function into which it enters and by the demand for the services of improved property in its location. Given the expectations of the future demand for the

The author would like to thank Mason Gaffney and C. Peter Rydell for valuable comments.

[1] Richard T. Ely and George S. Wehrwein, *Land Economics* (New York: The Macmillan Company, 1940), pp. 148–150.

[2] The marginal social product of land may differ considerably from the private marginal product if there are externalities associated with its use. To some extent these externalities are taken into account by zoning and other planning regulations. To the extent that these externalities are not accounted for by the planning framework, the optimal timing conditions presented here will differ from socially optimal conditions, for the present analysis is concerned with optimal timing from the point of view of the investor.

services of improved urban property, a production function will relate the inputs of capital and land to the output services of real estate in value terms: $Q = f(K, L)$, where $K =$ capital and $L =$ land. Although the service of real estate will be a flow through time, Q can be interpreted as the present value of this flow.

If there are diminishing returns to capital within the relevant range, the profit maximizing outlay on improvements to the site will occur where the marginal revenue product of an increment in capital investment in construction declines to the point where it equals the cost of the increment in capital investment in construction. The classic example of calculating the marginal revenue product of improvements to land is the determination of the optimal economic height of a skyscraper. As building height increases, structural elements, mechanical equipment, and elevators increase as a proportion of rental space, causing the price per square foot of rental space to increase. As the price per square foot increases with height, there comes a point where the cost of additional space is just equal to the discounted value of its future rental returns.[3] Though the example is most obvious in relation to building height, the method also applies to any sort of capital expenditure for improvements to land.[4]

Both the average and marginal revenue products of capital as a function of the capital input (K) to a fixed site are illustrated in Figure 1. In this figure, the average and marginal revenue products are calculated net of operating and maintenance expenses, and the optimum input of capital is $\bar{K}$. The resulting value of the improved site would be $OBC\bar{K}$. Subtracting the cost of construction, $OAD\bar{K}$, from the total property value will give the value of the bare site, $ABCD$, as a residual. This value formation process is implied by the real estate term "highest and best use," which refers to the form of development that would justify the highest payment for the site, whether as ground rent or as purchase price. If there is sufficient competition among potential users of the site, its market price should equal its value in the highest and best use, provided the optimal time for development is the current time.[5]

THE DYNAMIC CASE

But in a changing community, the optimal time for development may not be the current time. Population growth and redistribution, growing income, technological change, etc., will continually alter the demand for the services of real estate in specific locations. Thus, the APK and MPK curves will be shifting through

[3] Alfred Marshall called this point the "margin of building." *Principles of Economics* (8th ed.; London: Macmillan Ltd., 1961), p. 371.

[4] Alternatively stated, the profit maximizing outlay on improvements occurs where the marginal rate of return on capital investment in construction equals the interest rate.

[5] This description of the static case follows, for instance, that of Ralph Turvey, *The Economics of Real Property* (London: George Allen & Unwin Ltd., 1957).

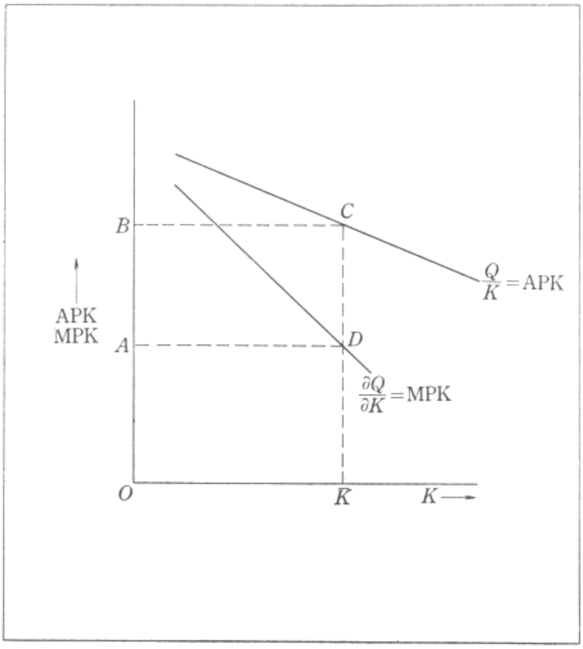

FIGURE 1

time, and the highest and best use of a particular site will consequently change.

In a growing community, there should be an increasing demand for the services of real estate, and this increased demand will be reflected in the demand for vacant sites by an upward shift of the APK and MPK curves of Figure 1. In addition to the effect of general community growth (or even in the absence of it), in particular areas the construction of new highways or other public or private investments may increase the demand for the services of real estate. Thus, as population and income grow, and as new infrastructure is created, increasing demand will justify more intensive use of the land, with larger construction outlays on the remaining vacant sites being called for. When the APK and MPK curves are gradually shifting upward as time passes, what is the optimum construction outlay, and when should the land be developed? The optimality criterion remains the same—that the present value of the land's marginal product be maximized. The problem is that, once the land is developed, the construction outlay usually freezes the land into a particular use and fixes for a long time the services that the property will render. For most uses, building codes, building technology, zoning regulations, and consumer tastes result in the construction of very durable structures. Thus, the development process usually involves the commitment of both land and capital to a long-lived combination, especially when high rise construction is involved. Further, quite aside from the great durability and immobility of most improvements, development tends to freeze land into a particular type of use merely by splitting the land into separate parcels; once subdivided into a large number of separate ownerships, land is difficult to reassemble for subsequent higher reuse. And the bundle of ownership rights to land can be subdivided even further, as between tenant and owner or between owner and lender, all of whose interests may conflict in regard to decisions about redeveloping improved land. These factors introduce an aspect of putty-to-clay irreversibility in allocation that can only be overcome by demolition or conversion, usually at great cost.[6]

The conclusion to be drawn from these observations on the rising demand for the services of improved property and the durability of improvements to land is that the appropriate form of development of a vacant site changes through time and that delaying the development of a site to a later date may result in a different (usually more capital-intensive) form of development, justifying a higher payment for the land. Thus, in the development timing decision, the value of a lower

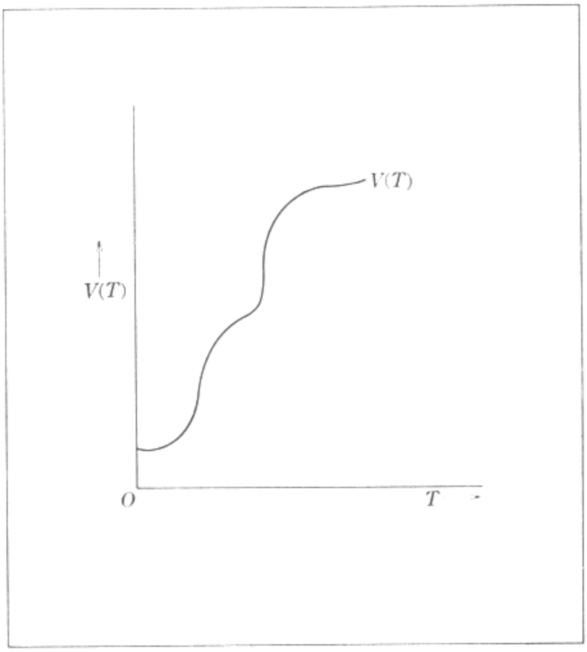

FIGURE 2

[6] Some buildings are even built with the intention of being potentially permanent. As an example, when the Singer Building in New York, once the tallest building in the world and still the tallest building ever to be demolished, was torn down recently, it was reported that the architect who designed the building stated that it was "designed to last as long as the pyramids." And as one example of the way that buildings face obsolescence in response to changed consumer tastes, the Singer Building had a central vacuum system with outlets in each executive office for cleaning silk hats.

intensity development justifying a lower payment for land now must be compared to a higher intensity development justifying a higher payment for the land at a later date.

To find the development date and capital intensity that maximizes the present value of the land, let $V(T)$ stand for the development value of the land at any future time *T if it is developed in its highest and best use at time T*. That is, $V(T)$ is given by the area *ABCD* in Figure 1, reflecting the optimal capital intensity if development were to take place at time *T*. But this development value changes through time because the APK and MPK curves are altered by the increasing demand for the services of improved property. For land on the fringe of an urban area, the path of $V(T)$ might take the shape of the curve in Figure 2. As population grows, previously rural land is demanded for urban uses, and $V(T)$ rises. The completion of highways, sewers, or other lumpy public or private facilities might cause a rapid or even discontinuous rise at some points; anything that affects the demand for the services of real estate in the vicinity of the site will affect $V(T)$. Naturally, unfavorable circumstances, such as the location nearby of a nuisance land use, may cause the development value of some land to decline, for the process is not one-way.

Given the curve of $V(T)$, the method of determining the optimal time of development closely resembles Wicksell's famous solution for the optimal period of maturing wine or for the optimal period of growth before harvesting trees.[7] The maximand is the present value (at time zero) of the development value of the land. If there are no operating expenses and no interim rent receivable for use of the land in the period preceding the time of development, the present value (at time t) of the land, $P(t, T)$, for any future development date, T, is given by the formula

$$P(t, T) = V(T)e^{-r(T-t)}, \qquad t < T \tag{1}$$

where r is the (instantaneous) rate of discount applicable in the real estate market.

To maximize the present value with respect to date of development, set

$$\begin{aligned}\frac{\partial P(t, T)}{\partial T} &= \frac{\partial}{\partial T}[V(T)e^{-r(T-t)}] = 0\\ &= V'(T)e^{-r(T-t)} - rV(T)e^{-r(T-t)} = 0 .\end{aligned}$$

Therefore,

$$\frac{V'(T)}{V(T)} = r . \tag{2}$$

Thus, the land should be developed when the rate of increase of the development value, $V(T)$, of the bare site equals the discount rate (this date is, of course,

[7] Knut Wicksell, *Lectures on Political Economy*, E. Classen, trans. (London: Routledge and Kegan Paul Ltd., 1934), I, pp. 178–181. $V(T)$ as defined here is analogous to the lumber content of a growing tree whose optimal date of harvest is to be determined.

not necessarily easy to identify in practice).[8] The second order condition must also be fulfilled.[9] It is that

$$\frac{V'(T)}{V(T)} = \frac{V''(T)}{V'(T)} .$$

If we let T_0 signify the optimal date of development thus determined, the present value of the land at any prior time, t, is

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)} , \qquad t < T_0 \tag{3}$$

and the rate of increase in the capitalized value of the land is

$$\frac{P'_t(t, T_0)}{P(t, T_0)} = r , \qquad t < T_0 . \tag{4}$$

This implies that, with unchanged expectations, we should expect unused land awaiting development to a higher use to appreciate at a rate equal to the interest rate at which future values are capitalized in the real estate market.[10] This result must be modified, however, to take into account the effect of ad valorem property taxes which are (or ought to be) levied on land as a fixed proportion of its market value. Let us assume that the tax is levied each period as a fixed percentage, a, of the market value of the land in that period, and that the development value of the land, $V(T)$, is figured after all taxes. Then equation (1) is altered to

$$P(t, T) = V(T)e^{-r(T-t)} - \int_t^T aP(i, T)e^{-r(i-t)}di . \tag{5}$$

To find the optimal development date in this case, set

$$\frac{\partial P(t, T)}{\partial T} = V'(T)e^{-r(T-t)} - rV(T)e^{-r(T-t)} - aP(T, T)e^{-r(T-t)}$$
$$- \int_t^T a\left[\frac{\partial P(i, T)}{\partial T}\right]e^{-r(i-t)}di = 0 . \tag{6}$$

From equation (5), at the optimal time of development the market price of the land must equal its development value

$$P(T, T) = V(T) ,$$

and also

$$\frac{\partial P(i, T)}{\partial T} = 0 \quad \text{(the condition for optimum } T\text{)}.$$

---

[8] Equations (1) and (2) show that this optimal date of development maximizes the present value of the land for all $t < T$.

[9] There is also the possibility of nonunique solutions, in which case the global maximum is the relevant one.

[10] The result is, of course, expected. Since appreciation in value is the only return from holding vacant land, the rate of appreciation should equal the rate of return on comparable alternative investments.

Therefore, from equation (6)

$$V'(T) - rV(T) - aV(T) = 0\,,$$

and

$$\frac{V'(T)}{V(T)} = r + a\,. \tag{7}$$

Thus, the effect of property taxes on vacant land is to bring development when the rate of increase of the development value, $V'(T)/V(T)$ of the bare site equals the sum of the interest rate and the tax rate. It is interesting that the difference between this optimal development timing condition and the one found previously in the absence of a land tax is one sense in which even a pure site value tax may not be perfectly neutral in its effect on resource allocation, as is frequently claimed.

The present value of the land at any time t prior to development is now

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)} - \int_t^{T_0} aP(i, T_0)e^{-r(i-t)}di\,. \tag{8}$$

Therefore,

$$\frac{\partial P(t, T_0)}{\partial t} = rV(T_0)e^{-r(T_0-t)} + aP(t, T_0) - r\int_t^{T_0} aP(i, T_0)e^{-r(i-t)}di$$

and the rate of increase in the value of the land is

$$\frac{P_t'(t, T_0)}{P(t, T_0)} = r + a\,, \qquad t < T_0\,. \tag{9}$$

Thus, the effect of the property tax on vacant land is to *decrease* the value of the land but to *increase* its rate of appreciation.[11] This could have been expected, because appreciation, the only return from holding the land, must now cover both the interest cost and lost taxes.

A further complication must also be mentioned. This is that land awaiting development or redevelopment to a higher use may be put to uses that will yield an interim income, either from agriculture, from an existing or remodeled lower-valued urban use, or from an intentionally temporary new improvement to the land that is meant ultimately to be removed to make way for a more permanent improvement (such temporary improvements are called "taxpayers" in real estate terminology). If a temporary improvement is contemplated, its form must be suited to the length of time it is expected to be used until the land is redeveloped. For short periods, the appropriate decision may be to make no temporary improvement at all, but rather to leave the land in its current use, or perhaps no use at all; this would be the case if no form of improvement would permit full amortization over

[11] This assumes that the level of property-related government services would be unchanged even in the absence of real estate taxes. If, however, the tax revenues were used to finance services of benefit to the property taxed, the value of the land might not decline at all.

its short lifetime. But as longer holding periods awaiting redevelopment are considered, some forms of short-term improvements will no doubt be feasible; for instance, paving for a parking lot, construction of advertising signs, or even temporary structures. For any given period of temporary use, the profit maximizing developer would choose that form of temporary improvement which would yield the highest present value of interim net rental returns, with the knowledge that the improvement would have to be removed at the end of the period.

Let $A(i, T)$ stand for the interim income of the land in each year, i, for the profit maximizing temporary use beginning during the current period (time 0) and ending in period T, the year of redevelopment of the land to a future higher use.[12]

Then, the present value of the land at time zero, $P(O, T)$ is found as before,

$$P(O, T) = V(T)e^{-rT} - \int_0^T aP(i, T)e^{-ri}di + \int_0^T A(i, T)e^{-ri}di\,. \tag{10}$$

The optimal date, T_0, of redevelopment to a higher use is again determined as in equation 6. Set

$$\frac{\partial P(O,T)}{\partial T} = V'(T)e^{-rT} - rV(T)e^{-rT} - aP(T, T)e^{-rT} - \int_0^T a\frac{\partial P(i, T)}{\partial T}e^{-ri}di + \int_0^T \frac{\partial A(i, T)}{\partial T}e^{-ri}di + A(T, T)e^{-rT} = 0\,.$$

As before,

$$P(T, T) = V(T) \text{ and } \frac{\partial P(i, T)}{\partial T} = 0\,.$$

Therefore,

$$V'(T) - rV(T) - aV(T) + e^{rT}\int_0^T A_t'(i, T)e^{-ri}di + A(T, T) = 0\,,$$

and the optimal date for redevelopment occurs when

$$\frac{V'(T)}{V(T)} = r + a - \frac{A(T, T) + e^{rT}\int_0^T A_t'(i, T)e^{-ri}di}{V(T)}\,. \tag{11}$$

The economic interpretation of equation (11) is that development to a higher use should take place when the rate of change of development value of the land, $V'(T)/V(T)$, is equal to the sum of the interest rate and the tax rate, minus the rate of return in interim use earned by delaying the date of redevelopment one more period.[13] The rental return from interim use earned by delaying the date of develop-

[12] It will be assumed that the development value of the land, $V(T)$, is independent of the interim use to which the land is put. Any cost of removing interim improvements is deducted from $A(T,T)$.

[13] Note that this does *not* say that redevelopment of improved property to a higher urban use should be undertaken as soon as the development value of the site, net of clearance costs, exceeds the value of the existing improved property, as is sometimes stated.

ment one more period (the numerator of the last term) is composed of two parts: (1) the rental in period T and (2) the cumulated value at time T of the increase (if any) in rental returns from interim use associated with a lengthening of the planned lifetime of the interim improvement by one period. The explanation for the second part is that a longer-lived interim use might also be a more capital-intensive use, with higher site rental over the whole interim use interval. For short interim use intervals with no interim improvement, this second term of the numerator would be zero.

The present value of the land at any time, t, prior to development becomes

$$P(t, T_0) = V(T_0)e^{-r(T_0-t)} - \int_t^{T_0} aP(T_0)e^{-r(i-t)}di + \int_t^{T_0} A(i, T_0)e^{-r(i-t)}di\,, \quad (12)$$

and the rate of appreciation is

$$\frac{P'(t, T_0)}{P(t, T_0)} = r + a - \frac{A(t, T_0)}{P(t, T_0)}\,, \qquad t < T_0\,. \quad (13)$$

That is, the rate of appreciation of land awaiting development to a higher use is the sum of the interest rate and the tax rate, minus the rate of return earned in any interim use in that period. This result follows strictly from the assumption that the value of the land is determined by capitalization of future returns. The curves of $V(T)$ and $P(t, T_0)$ could look something like those shown in Figure 3, and

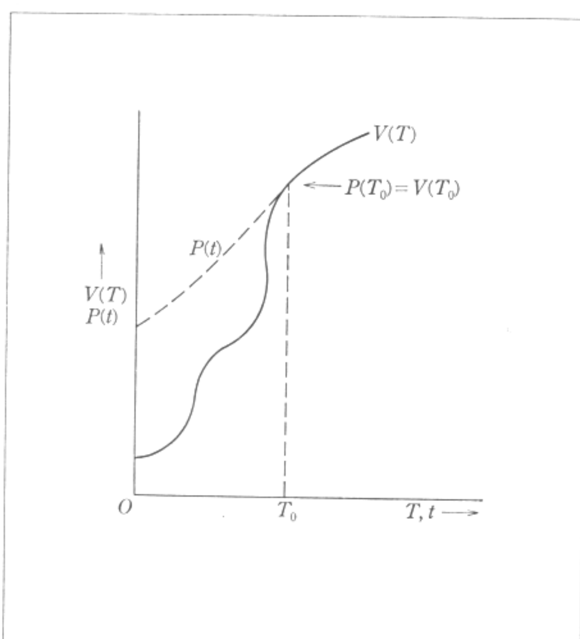

FIGURE 3

the optimal date of development would be at T_0.[14]

If there were perfect competition and perfect information in the real estate market, the market price of the site (at any time prior to T_0) would be $P(t, T_0)$, and the site would appreciate at the rate given by equation (13). The market price of the site can be substantially above its value in whatever current use it may have, and above the value it would have even if it were developed for its current highest and best use (that is, $V(T)$). Such behavior is familiar with land on the fringe of urban areas, where price is determined primarily by future development potential, rather than by any return in current use. The land may be held by farmers, estate owners, speculators or others, but whoever holds it serves the function of preserving the land from premature development when expected increases in demand would justify more capital intensive development at a later date. Unless their actions are based on incorrect forecasts or tend to be destabilizing, speculators can contribute to the efficient working of the real estate market by bearing the risks and costs of holding land during the period of transition between rural and urban use, or between a lower and a higher urban use. However, note that this is essentially a short-run model applying to land in the process of transition from one use to another, and has little application to very long-run or static situations.[15] Also, it is a partial equilibrium model, referring to individual sites in a given (though changing) environment. Finally, the only form of taxation incorporated in this model is the property tax, and a more complete model would have to take into account the effects of other relevant forms of taxation.

UNCERTAINTY AND MARKET IMPERFECTIONS

Obviously, the smooth and efficient operation of the real estate market along the lines described makes extraordinary demands on information about the future. The conventional textbook application of the Wicksell model to determine the optimal date for cutting trees relies on well-established biological growth rates and some assumption about future lumber prices to predict the future time path of $V(T)$. But, in the real estate market, investors will not have certain or uniform expectations concerning the future course of $V(T)$, and so there will be no firm

[14] A recent study found that between 1954 and 1962, vacant land awaiting urbanization in one area of Philadelphia appreciated at a rate of 9.7 per cent per year in real terms when several factors (location, state of land, zoning) were held constant. Real estate taxes reduced this return to 7.7 per cent per year, which the authors felt represented a normal rate of return considering the risk, illiquidity, and transaction costs involved in land investment. F. Gerard Adams, Grace Milgram, Edward W. Green, and Christine Mansfield, "Undeveloped Land Prices During Urbanization: A Micro-Empirical Study Over Time," *Review of Economics and Statistics*, L, No. 2 (May, 1968), pp. 248–258.

[15] Theoretically, with perfect certainty and an infinite time horizon, an investor would plan a *sequence* of developments for the site. Practically, the typical building lifetime is probably so long in relation to the feasible time horizon that investors would not plan beyond the lifetime of the next improvement.

idea of the optimal date of development, T_0, or of the development value of the land at that date, $V(T_0)$. At best, investors will have some joint probability distribution for $V(T_0)$ and T_0 which will alter as time passes and more information becomes available about the speed, direction, and character of new urban development. Thus, though the capitalization process theoretically underlies the formation of market price for land awaiting new development, the actual price of land, $P(t)$, cannot be expected to show the smooth upward path that differentiation of the simple present value equation suggests. Particularly, as more information about prospective development patterns becomes known, the price path of land can be expected to shift up or down discontinuously according to how the information affects investors' expectations, and speculative gains or losses will occur.

Since the uncertainty surrounding $V(T_0)$ usually is greater when T_0 is more distant, the discount rate used to arrive at the present value of land awaiting new development may be higher earlier in the period before development, decreasing as T_0 nears. If this is the case, one would expect the land to appreciate more quickly in the earlier stages of the period prior to development, other things the same. Also, uncertainty about the future demand for real estate in an area would lead to a desire for flexibility in improvements. In particular, it may discourage permanency as an aspect of construction in favor of buildings with a lower initial capital outlay and higher maintenance costs so that in case a building is scrapped for redevelopment of the site there will be a smaller capital loss.

Finally, the simplicity of the capitalization process is marred by the many imperfections of the real estate market: land is infrequently transacted, transactions costs are high, there is limited dissemination of transaction prices, land is not homogeneous, and special needs of buyers and sellers enter; all of which will lead to many errors of judgment based on imperfect knowledge of the future.[16] However, the resulting dispersion of market prices about an upward trend is not, in itself, inconsistent with the capitalization process, but is, instead, a reflection of the inevitable uncertainty and the many imperfections of the real estate market.

CONCLUSION

A modified Wicksell model has been used to explain optimal timing decisions concerning the development of land from a lower to a higher urban use. The analysis has shown that, within the context of the assumptions, the optimal date for development or redevelopment of urban land depends on (1) the discount rate applying in the real estate market, (2) the property tax rate, (3) the earnings in any interim use, and (4) the way in which the highest and best use of the land is expected to change in the future. The analysis has also shown that the mere presence of rises in the value of land awaiting development to a higher use should not automatically be interpreted as speculative gains; rather, an increase in the value of such

[16] The earlier mentioned externalities associated with land use are a different sort of imperfection which would lead to nonoptimal allocation even in the absence of other market imperfections.

land is inherent in the process of capitalization of future income. More properly, only "windfall" changes in the value of land, which arise because of imperfectly foreseen events that affect future development potential, should be considered speculative gains.